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# "Damage" in the Low-Temperature Phase of the $\pm J$ Spin Glass in Two to Six Dimensions

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Identical clones are made of configurations of the $\pm J$ spin glass at temperatures near and below the phase transition, and their damage is monitored as a function of temperature and time. Focal points with multiple bifurcations are found occurring near the onset of the phase transition temperature.

**KEY WORDS:** Spin glass; damage; Monte Carlo simulations; replicas.

The spin-glass problem has attracted a wide variety of methods, each with varying degrees of success.[1-8] Recently we have interpreted the onset of the spin-glass phase transition as a dynamic percolation problem.[9] The flipping probabilities $p$ of the various sites are measured and the threshold value of $p$ determined where for the first time there is a spanning cluster in our system. The variation of this percolation threshold $p_c$ shows a monotonic decrease with temperature. We also introduced a somewhat *ad hoc* definition of "freezing" and the intersection of the "freezing" curve with the threshold curve is in good agreement with the numerically determined phase transition temperatures. This approach was modified by Stevens *et al.*[10] for the five- and six-dimensional spin glasses and also for monitoring the variation of the phase transition temperature with the dilution of bonds for three-dimensional systems.

In this work we continue our investigation of the spin-glass problem using a method which is based on the early work of Binder and Schroeder[11] and also elaborate on ideas developed in earlier studies.[6,12] We simulate the $\pm J$ spin glass in two to six dimensions using the standard heat-bath Monte Carlo algorithm.[13] The bonds of the lattice are randomly selected

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as ferromagnetic or antiferromagnetic with a probability of 0.5 at the onset of the simulation and all the clones share the same bond configuration with their parents and with each other. We monitor the evolution of the damage[2] between two initially identical configurations as a function of temperature and time. The damage at a site is equal to one if the two spins at that site for the replicas being considered are in opposite states and equal to zero if they are in the same state. The damage between the two replicas is the total damage divided by the number of sites in the system. An equilibrium configuration at temperature $T$ is cloned to produce an identical replica and the two systems are allowed to evolve subject to the same dynamics, but they both interact independently with the thermal reservoir, i.e., different random numbers are used in the updates. If the cloning is made at a high temperature, then we expect the two systems to evolve along separate and independent paths in phase space and to have an average "damage" per site of 0.5. At low temperatures a clone will be expected to have a strong overlap with its "parent" and hence a damage closer to zero. The damage is similar to the Binder–Parisi order parameter $\Psi = \sum_i S_i^\alpha S_i^\beta$, where $\alpha$ and $\beta$ refer to different replicas. We emphasize here that we are not measuring thermodynamic quantities such as the correlation length and therefore are not concerned with the damage difference.[15,16]

As a test for our program we performed an equivalent simulation of the Ising model and observed that for clones constructed below $T_c$ the damage is close to zero and for clones constructed in the high-temperature phase and cooled to low temperatures they had a damage close either to zero or one. The bifurcation of the clones into the two attractors occurs at the critical temperature for the ferromagnetic system. We summarize these results: for clones constructed above $T_c$ and for simulations performed at this temperature the damage is close to 0.5, while for clones constructed below $T_c$ and simulations done at this low temperature, the damage is close to 0. For clones constructed above $T_c$ and simulations done at low temperatures there is equal probability for the clone to have a damage close to zero or close to one.

We present our results for the $d$-dimensional spin glasses and then make our somewhat speculative statements below. We apply the standard heat-bath Monte Carlo dynamics to our model. The sites are visited in a random manner, but each site is visited once in a Monte Carlo step (MCS). At each temperature a clone is made of the most recently created clone and the systems are all allowed to equilibrate for 8000 MCS and then the damage is measured between each pair of replicas. This measurement

[2] See ref. 14 and references therein for earlier work related to damage and the spin glass.

is made for about 400 samples where each sample is taken after every 30 MCS. The systems are now prepared for the start of the next lower temperature and a clone is made of the one recently constructed at the higher temperature and the procedure outlined above is repeated.

For each dimension we display two parts to each figure. Part a shows as a function of temperature the overlap between all the clones and the primary parent. The first few siblings are created above the spin-glass temperature and the subsequent dependents are created from other siblings. The same simulation performed on the Ising model would lead to a bifurcation of the damage from 0.5, at and above $T_c$, to values which either move toward zero or toward one with decreasing temperature. For the two-dimensional spin glass of linear dimension $L = 128$, we observe a focal point, i.e., a confluence of all bifurcations at $T = 1.0J/k$, and the damage spreads out with decreasing temperature more or less forming the shape of a fan (Fig. 1). The equivalent data for the three-dimensional system show a focal point at about $1.7J/k$ but a clear separatrix, in that there is no damage between the primary parent and all the siblings with a value near to 0.5, but it occurs at about 1.1 for a system of linear dimension $L = 32$. At this temperature the clones separate into two major branches with the region arround 0.5 empty. Again the same behavior is observed for the four-dimensional system of linear dimension $L = 12$. The focal point is at $T = 2.25$ and the vacant region appears at about 2.2, as seen in Fig. 2a. The five-dimensional data show the same typical behavior as the four- and three-dimensional systems and here the focal temperature is at 2.6, which is also the apparent start of the separatrix region. The focal temperature for the six-dimensional system (Fig. 3) is about $T = 3.0$, but note here that our system size linear dimension is only 6. We find the absence of damage at or near 0.5 at low temperatures somewhat surprising, as naively the presence of a multiplicity of ground states should lead one to expect that the damage for some of the clones should be close to 0.5. This result may reflect that there are only a few replicas created above $T_c$, the spin-glass transition temperature, and those created at lower temperatures are restricted in phase space. This effect requires further investigation. The onset of the separatrix region is in better agreement with the series expanion estimates of $T_c$,[17] whereas the onset of the bifurcations is at a higher temperature.

Figures 1b, 2b, and 3b show the overlap between the newly created clone and its "parent" as a function of temperature. This stays at about 0.5 until we reach the focal point, then it monotonically decreases with temperature to zero. We also monitor the damage between a parent and its clone which is created at some finite temperature as a function of subsequent cooling. These appear to converge to nonzero values as we reduce the temperature toward zero. We conclude from these data that only at

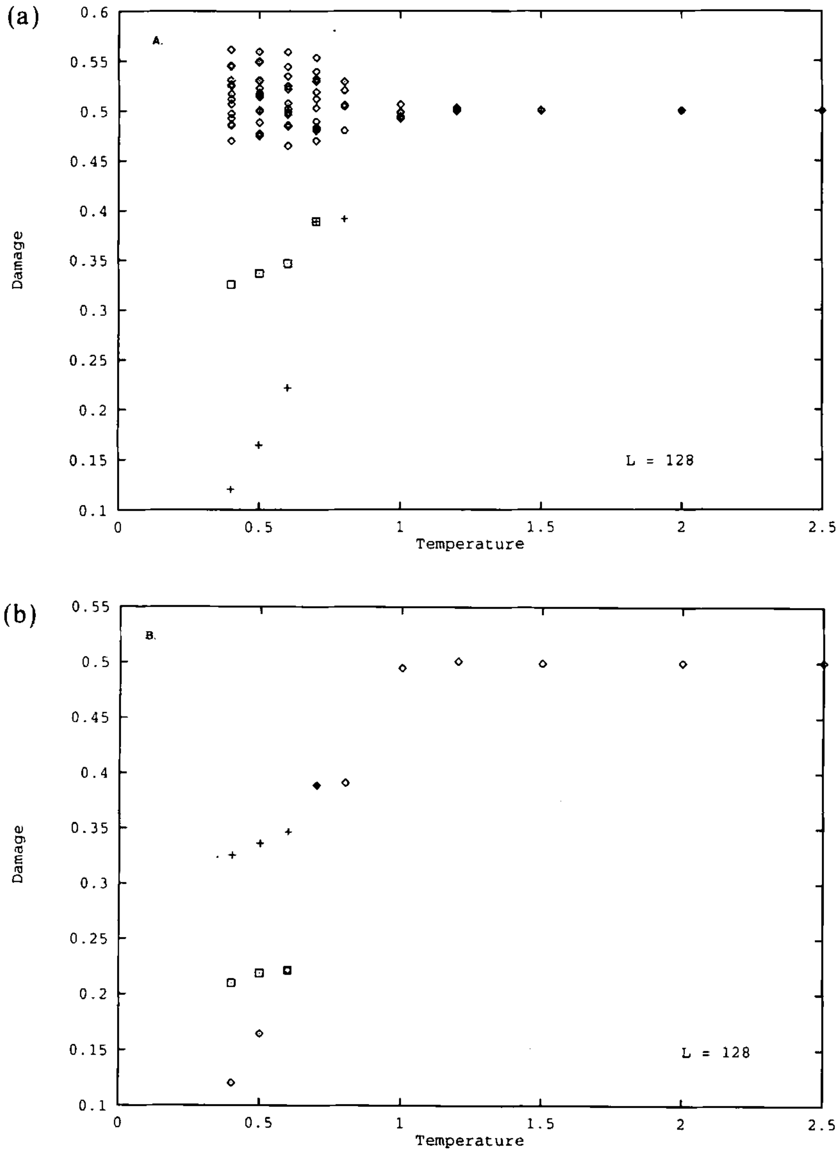
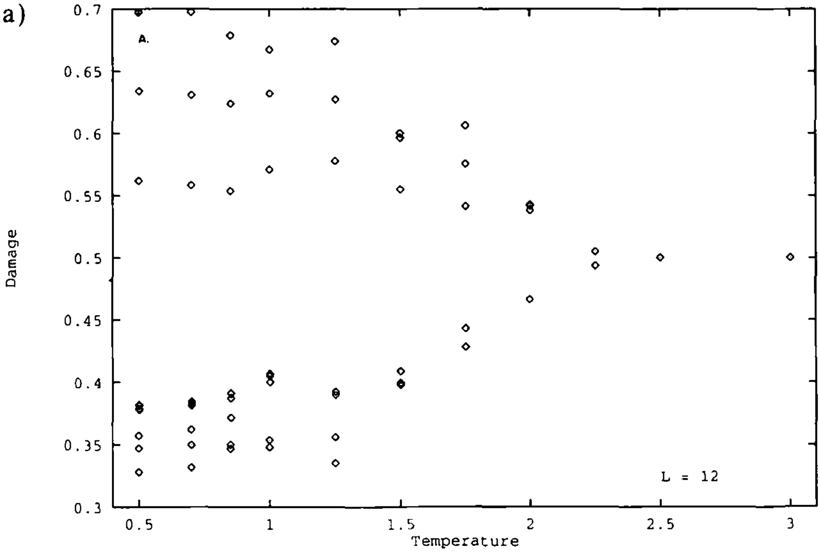



Fig. 1. (a) The "damage" between the primary parent and all the clones vs. temperature for the two-dimensional system. There is a focal point for the "fanlike" low-temperature behavior at $T$ near 1.2. (b) The "damage" between the most recently created clone and its parent is shown by diamonds. The damage between the clone created at $T = 0.7$ and its parent is shown by plusses and between the clone created at $T = 0.6$ and its parent is shown by squares.

(a)




(b)

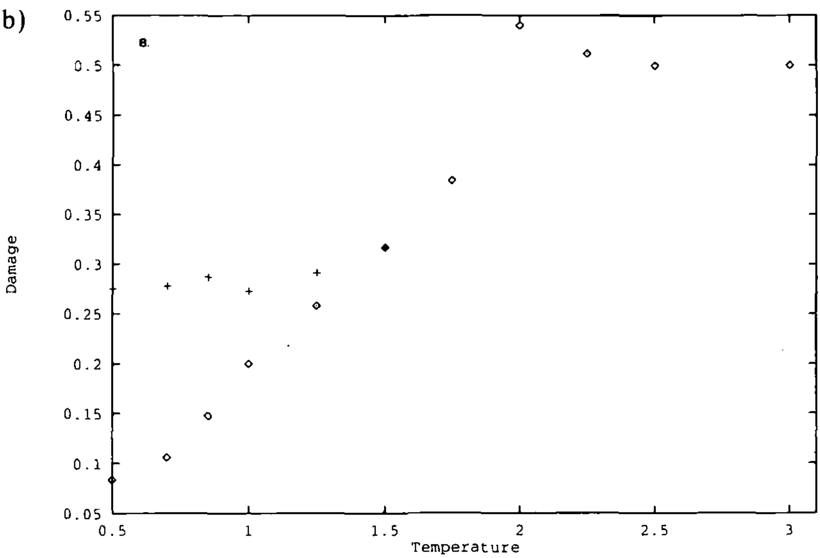
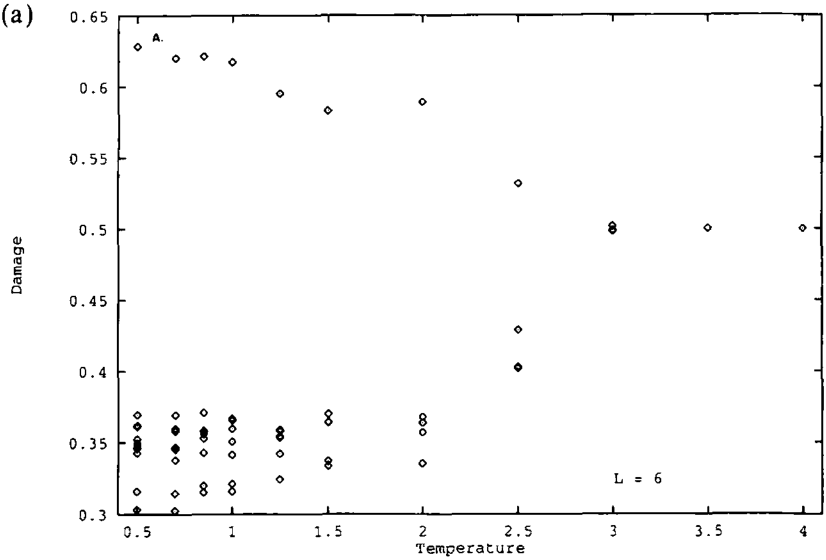



Fig. 2. (a) The same as Fig. 1a, but now for the four-dimensional spin glass. The focal point of the fan is at $T = 2.2$. (b) The diamonds represent the same information as in Fig. 1b and the plusses give the "damage" between the parent and clone created at $T = 1.5$.

(a)




(b)

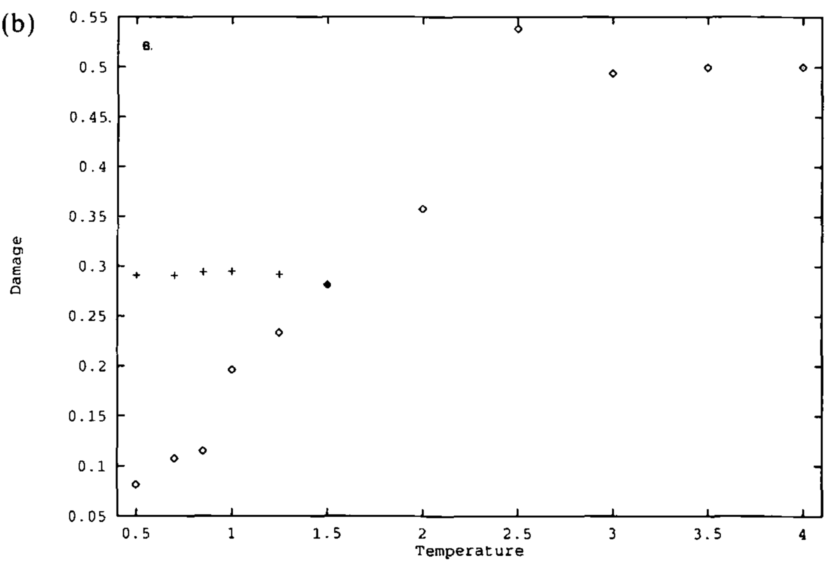



Fig. 3. (a) The same as in Fig. 1a, but for the six-dimensional spin glass. The focal point for the "fan" is at $T = 3.0$. (b) The diamonds represent the same information as in Fig. 1b. The damage between the clone created at $T = 1.5$ and its parent as a function of temperature is shown by plusses.

very low temperatures is it possible for replicas to be found close to each other in phase space. Replicas created below the spin-glass transition temperature still maintain a finite and near constant damage as they are further cooled to zero temperature.

In this way, we have monitored the evolution of a system in phase space at different temperatures and have observed the structure of the low-temperature spin-glass phase on the trajectories. All our observations are subject to the standard reservations for Monte Carlo simulations—finite system size, finite time, equilibration concerns, choice of random numbers, etc. We find that the damage is 0.5 when the systems are above the expected spin-glass temperature, while there is a "fanlike" deviation below this temperature. This "fanlike" property with a focal point is also observed in two dimensions at about $T=1$ which in the past had been mistakenly identified as a transition temperature. We monitor the damage between the primary parent and all the clones and these show for the three- to six-dimensional systems an apparent separatrix region about the damage of 0.5 which gets wider with temperature. We have also monitored the damage between the most recently created clone and its parent and this shows the expected decrease toward a value of zero at low temperatures. Somewhat surprising is the fact that a clone created at a low temperature maintains its damage with its parent as the temperature is further decreased. This perhaps is an indicator that there are barriers at all heights in addition to rough valleys. We have repeated some of the simulations with different bond configurations and the results found above are reproducible.

There is a close relationship between the "freezing" ideas outlined in the introduction and the "damage" approach we have considered here. The damaged sites on average have more bonds unsatisfied. These are the unfrozen sites. We define a "frozen" site as one whose nearest neighbor environment remains more or less unchanged for the duration of the simulation and also where the majority of bonds are satisfied. A picture is emerging in which the spin glass may be viewed as the evolution of frozen islands into larger structures as the temperature is decreased. At the transition temperature the system has a spanning cluster of "frozen" sites. The intriguing point is that given a "frozen" structure, when a clone is made, subsequent cooling still leads to distinct newly created frozen structures as shown by the finite damage maintained between the parent and its clone. This damage is much greater than the damage between the clone created at the lower temperature and its parent. In the future we propose to monitor the overlap between the "satisfied" or strongly bonded spins in the various clones and how this number increases as the temperature is decreased. This should shed further light on the nature of the low-temperature phase of the spin glass and the relevance of the mean-field[1] and "droplet"[4] pictures to finite-dimensional systems.

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